de Haas-van Alphen effect in strongly correlated electron systems

I. Sheikin
Grenoble High Magnetic Field Laboratory
CNRS, Grenoble, France
1. Introduction to de Haas-van Alphen effect
   - Historical aspects
   - Landau quantisation
   - Lifshits-Kosevich formula
   - Fermi surface
   - Effective mass
   - Dingle temperature

2. Experimental technique – torque magnetometry
3. Strong electronic correlations and dHvA effect
   - $f$-electrons: itinerant or localized?
   - Phase transitions and Fermi surface
   - Spin-splitting and effective mass
   - Field-dependent effective mass
   - Field-dependent frequencies

4. Summary
de Haas-van Alphen Effect - discovery

W. J. de Haas (1878-1960)  
P. M. van Alphen (1906-1967)

Oscillations of magnetization in bismuth in magnetic field  
Comm. Phys. Lab. Leiden, No. 212a (1930)
Landau quantisation

Quantisation of the orbital motion of a charged particle in a magnetic field (1930)

\[ E = (n + 1/2)\hbar\omega_c, \quad \omega_c = \frac{eB}{mc} \]

Allowed orbits are in the plane perpendicular to the magnetic field direction on a series of constant energy surfaces in \(k\)-space, known as Landau tubes.
Lifshits-Kosevich formula (1954)

I. M. Lifshits (1917 - 1982)

A. M. Kosevich (1928 - 2006)

\[ M_{osc} = \sum_r \sum_i \frac{(-1)^r}{r^{3/2}} A_i \sin \left( \frac{2\pi r F_i}{B} + \beta_i \right), \quad F_i = \frac{\hbar c}{2\pi e} S_i \]

- \( F_i \) - oscillatory (dHvA) frequency,
- \( S_i \) - Fermi surface extremal cross-section

\[ A_i \propto B^{1/2} \left| \frac{\partial^2 S_i}{\partial k_H^2} \right|^{-1/2} R_T R_D R_S \]

is small for cylindrical FS, giving large oscillations
is large for pancake-like FS, giving small oscillations
dHvA effect and Fermi surface topology

\[ \tilde{M}_i \propto A_i \sin \left( \frac{2\pi F_i}{B} + \beta_i \right), \quad F_i = \frac{\hbar c}{2\pi e} S_i \]

Fermi surface topology:

\[ F(\theta) = \frac{\hbar c}{2\pi e} S(\theta) \]

Spherical Fermi surface
Extremal cross-section does not depend on the field orientation

\[ S(\theta) = \text{const} \]
\[ F(\theta) = \text{const} \]

2D Fermi surface

\[ S(\theta) = S_0 / \cos \theta \]
\[ F(\theta) = F_0 / \cos \theta \]
Temperature damping factor and effective mass

\[ A_i \propto B^{1/2} \left| \frac{\partial^2 S_i}{\partial k_H^2} \right|^{-1/2} R_T R_D R_S \]

\[ R_T = \frac{\alpha m^* T / B}{\sinh(\alpha m^* T / B)} \]

\[ \alpha = 2\pi^2 c k_B / e\hbar \approx 14.69 \text{ T/K} \]

\[ m^* = \frac{\hbar^2}{2\pi(\partial S_F / \partial E)} \] - effective mass

![Graph showing temperature damping factor and effective mass vs. temperature. The graph includes a curve for different temperatures and labels for 100\(m_0\), 1\(m_0\), and 4\(m_0\) with corresponding temperature ranges.](image)
Effective mass - determination

Temperature dependence of the oscillations amplitude:

\[ A(T) = A_0 \frac{\alpha m^* T / B}{\sinh(\alpha m^* T / B)} \]

- Plot
\[ \ln\{A[1 - \exp(-2\alpha m^* T / B)] / T\} \] vs. \( T \)

- Vary \( m^* \) to obtain the best linear fit

Mass plot

Slope is proportional to the effective mass
Dingle damping factor and Dingle temperature

\[ A_i \propto B^{1/2} \left| \frac{\partial^2 S_i}{\partial k_H^2} \right|^{-1/2} R_T R_D R_S \]

\[ R_D = \exp(-\frac{\alpha m^* T_D}{B}) \]

\[ \alpha = \frac{2\pi^2 c k_B}{e \hbar} \approx 14.69 \text{ T/K} \]

\[ T_D = \frac{\hbar}{2\pi k_B} \frac{1}{\tau} \]

\[ \tau \text{ - scattering rate} \]

\[ l = \frac{\hbar^2 k_F}{2\pi k_B m^* T_D} \]

- mean free path

![Graph showing dHvA Amplitude vs. B (Tesla) for different temperatures: T_D = 0.1 K, T_D = 0.5 K, T_D = 1 K.](image)
Dingle temperature - determination

Field dependence of the oscillations amplitude:

\[ A(B) \propto B^{-1/2} \frac{\exp(-\alpha m^* T_D / B)}{\sinh(\alpha m^* T / B)} \]

\[ \ln(AB^{1/2} \sinh(\alpha m^* T / B)) = \text{Const.} - \alpha m^* T_D / B \]

Dingle plot

Dingle temperature is determined from the slope of the Dingle plot
Spin damping factor and spin-zeros

\[ A_i \propto B^{1/2} \left| \frac{\partial^2 S_i}{\partial k_H^2} \right|^{-1/2} R_T R_D R_S \]

\[ R_S = \cos\left( \frac{\pi g r m^*}{2 m_0} \right) \]

\[ g \text{ – Lande } g\text{-factor} \]
\[ r \text{ – harmonic number} \]

Oscillations amplitude vanish for:

\[ \frac{grm^*}{m_0} = 1 + 2n \]

Spin-zeros

If the \( m^*(\theta) \) is known, allows for the determination of the \( g \)-factor
de Haas-van Alphen effect - conditions

$$E = (n+1/2)\hbar \omega_c, \quad \omega_c = \frac{eB}{m^* c}$$

High fields and low temperatures

$$\frac{\hbar \omega_c}{k_B T} \gg 1$$

$$\frac{B}{T} \gg \frac{m^* c k_B}{\hbar e}$$

$$\omega_c \tau / 2\pi > 1$$

High quality samples
Experimental technique – torque magnetometry

In magnetic field:

\[ \vec{T} = \vec{M} \times \vec{B} \]

Bending of the cantilever

Variation of the capacitance, \( C \)

Assumption: \( \Delta C \propto T \) (for small deformations)

\[ T = M_{\perp} B V \]

For an anisotropic Fermi surface:

\[ \tilde{M}_{\perp} = \frac{1}{F} \frac{\partial F}{\partial \theta} \tilde{M}_{\parallel} \]
Strongly correlated electron systems

In some Ce-, U- and Yb-based intermetallic compounds, the electronic correlations (f – conduction electrons) are so strong that \( m^* \sim 10^2 – 10^3 \ m_e \) (heavy fermions).

Observation of Heavy Electrons in CeRu₂Si₂ via the dHvA Effect

Haruyoshi Aoki, Shinya Uji, Ariane Keiko Albessard\(^\dagger\) and Yoshichika Ōnuki\(^\dagger\)

\(\ddagger\)National Institute for Metals, 2-3-12 Nakameguro, Meguro-ku, Tokyo 153
\(\ddagger\)Institute of Materials Science, University of Tsukuba, Tsukuba, Ibaraki 305

(Received July 23, 1992)

We have observed a new de Haas-van Alphen (dHvA) oscillation in CeRu₂Si₂ of frequency \( 5.36 \times 10^7 \) Oe and cyclotron effective mass \( 120 m_0 \) for magnetic fields along the [100] direction. The new dHvA frequency is thought to arise from the heavy electrons of the large hole surface centered at the Z point which is predicted by the band structure calculations.

Conditions:

\[
\frac{B}{T} \gg \frac{m^* c k_B}{\hbar e}
\]

For \( m^* = 100 \ m_e \): \( B/T \gg 75 \ T/K \)

For a good dilution refrigerator with \( T_{\text{base}} \sim 20 \ \text{mK} \): \( B \gg 15 \ T \)
**Heavy fermion systems**

**Doniach phase diagram**

\[ T_K \propto e^{-1/J_n(E_F)} \]

\[ T_{RKKY} \propto J^2 n(E_F) \]

Competition between two interactions:

- **Kondo**
  - Screening of the localised magnetic moment by conduction electrons
  - Creates heavy quasi-particles.

- **RKKY**
  - Promotion of the long-range magnetic order.
  - Gives rise to magnetic ordering.

S. Doniach, Physica B 91, 231 (1977)

Comparison of the experimental and calculated Fermi surfaces:
- itinerant $f$-electrons $\Rightarrow$ big Fermi surface (calculated for Ce)
- localized $f$-electrons $\Rightarrow$ small Fermi surface (calculated for La)

Magnetically ordered Kondo systems:
- $X < X_c$: localized $f$-electrons
- $X > X_c$: itinerant $f$-electrons

... but high magnetic fields are required to study Fermi surfaces
CePd$_2$Si$_2$: AF Kondo compound

Body-centered tetragonal structure

Antiferromagnetic transition at $T_N \sim 10$ K

$k = [1/2,1/2,0]$, $m_{AF} // [110]$

Suppression of AF order at $P_c \sim 27$ kbar

dHvA oscillations observed in the basal plane only

Fairly good agreement between calculated and experimental dHvA frequencies!
Fermi surface for LaPd$_2$Si$_2$ (localized $f$-electrons)

No correspondence between calculated and experimental dHvA frequencies!

Itinerant $f$-electrons even at high field!
CeRh$_2$Si$_2$: metamagnetic transition

Two-steps metamagnetic transition @ ~ 26 Tesla

In resistance

In magnetic torque

Metamagnetic transition becomes strongly first order at low temperature.
Metamagnetic transition and Fermi surface

Both the size (dHvA frequency) and topology (dHvA amplitude) of the Fermi surface change dramatically across the metamagnetic transition.
Spin-splitting and effective mass (CePd$_2$Si$_2$)

- $\zeta$-branch is split in high field
- big difference in effective masses
Spin-splitting and effective mass (CeCoIn$_5$)

In good agreement with A. McCollam et al. 2005 (PRL 94, 186401)
Field-dependent effective mass

**Theoretical prediction**
(A. Wasserman *et al.*, J.P.C.M. 1, 2669 (1989))

\[
\frac{m^*}{m_b} = 1 + \frac{2Dn_f}{Nk_B T_K} \left(1 + Jg\mu_B B/k_B T_K \right)^2
\]

- Most of the effective masses decrease with field
- Non-monotonic field dependence of \(m^*(F_2)\) and are due to the spin-splitting
Field-dependent dHvA frequencies

Some frequencies are field-dependent.
Summary

• De Haas-van Alphen effect is a powerful tool to explore the electronic structure of metals.

• Additionally, in strongly correlated electron systems it allows for investigation of unusual features such as field-induced modifications of the Fermi-surface, field-dependent dHvA frequencies and effective masses, etc…